B3.2-R4: DISCRETE STRUCTURES

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**	
2.	**Parts of the same question should be answered together and in the same sequence.**	

Time: 3 Hours **Total Marks: 100**

1.

a) Show that the function $f(x) = x^4$ and $g(x)x^{\frac{1}{4}}$, for all real numbers x, are inverses of one another.

b) Show that the propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

c) In how many ways can five examinations be scheduled in a week so that no two examinations are scheduled on the same day considering Sunday as a holiday?

d) Find the generating function of the sequence 2, 3, 2, 3, 2, 3,... .

e) Draw the graph G corresponding to the following adjacency matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

f) Let σ and τ be the following elements of the symmetric group S_6.

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 5 & 4 & 6 & 2 \end{pmatrix} \qquad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{pmatrix}$$

Find $\sigma\tau$, σ^{-1}.

g) Describe the language L(G) generated by the following grammer:
$V_N = \{S\}, V_T = \{a, b\}$ with production $S \rightarrow a, S \rightarrow Sa, S \rightarrow b \text{ and } S \rightarrow bS$.

(7x4)

2.

a) Find the truth value of the following statement
$[p \rightarrow ((q \wedge (\sim r)) \vee s] \wedge [(\sim u) \leftrightarrow (s \wedge r)]$, when u is false and p, q, r and s are true.

b) Solve the following recursive relation using substitution:

$$f(n) = f\left(\frac{n}{2}\right) + 1, f(1) = 1$$

where n is an integer greater and equal to 1.

c) Let $R \text{ and } S$ be the following relations on the set A = {1, 2, 3}
$R = \{(1,1), (1,2), (2,3), (3,1), (3,3)\}$, $S = \{(1,2), (1,3), (2,1), (3,3)\}$
Find $R \, o \, S$. Is $R \, o \, S$ an equivalence relation on A? Give reasons.

(6+6+6)

3.

a) Consider the group $G = \{1, 2, 3, 4, 5, 6\}$ under multiplication modulo 7.
 i) Find the multiplication table of G.
 ii) Find $2^{-1}, 3^{-1}, 6^{-1}$.
 iii) Find the subgroups generated by 2 and 3 and also find their orders.
 iv) Is G cyclic?

b) Solve the following recurrence relation
$t_n = 4(t_{n-1} - t_{n-2})$, $n \geq 2$ and $t_0 = 1$, t1 = 1.

c) Negate the following statements:
 i) $\forall x \, \forall y \, \exists z, \, p(x, y, z)$
 ii) $\exists x \, \forall y \, \forall z, \, p(x, y, z)$
 iii) $\exists x \, \exists y \, \forall z, \, p(x, y, z)$
 iv) $\forall x \, \exists y \, \exists z, \, p(x, y, z)$.

(8+6+4)

4.

a) Find all x such that $1 \leq x \leq 100$ and $x \equiv 6 \pmod{13}$.

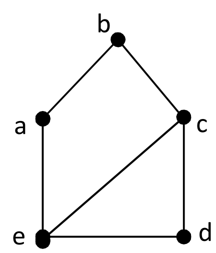
b) A word that reads the same when read in forward or backward is called a palindrome. How many seven- letter palindromes can be formed from 26 English alphabets?

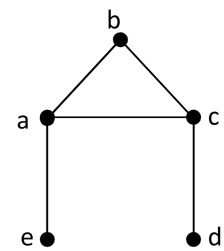
c) In a graph of 97 students, the number of students taking Computer Science is twice the number taking Mathematics. How many students are taking Computer Science?

(4+6+8)

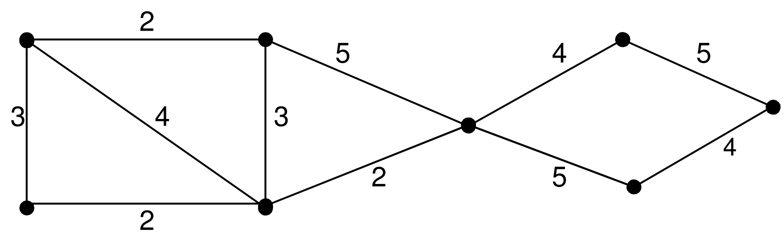
5.

a) Are the following graphs isomorphic? Why?





b) Find the minimal spanning tree for the following weighted connected graph:



c) Can a simple graph exist with 15 vertices, each of degree five? Give reason to support your answer.

(6+8+4)

6.

a) Find a minimal sum-of-products form for each of the following complete sum-of-products Boolean expressions:

i) $E_1 = xy + x'y + x'y'$;

ii) $E_1 = xy + x'y'$

b) Let M be the automaton with the following input set A, state set S, and accepting ("yes") state set Y :

$A = \{a, b\}, \ S = \{s_0, s_1, s_2\}, \ Y = \{s_1\}$

Suppose s_0 is the initial state of M, and next state function F of M is given by the following table:

F	a	b
S_o	S_0	S_1
S_1	S_1	S_2
S_2	S_2	S_2

i) Draw the state diagram $D = D(M) of\ M$.

ii) Describe the language $L = L(M)\ accepted\ by\ M$.

c) Let $a = 37\ and\ b = 249$.

i) Find $d = gcd(a, b)$.

ii) Find integers m and n such that d = ma + nb.

iii) Find lcm (a, b).

(6+6+6)

7.

a) Use mathematical induction to show that $1+\frac{1}{4}+\frac{1}{9}+...+\frac{1}{n^2}<2-\frac{1}{n}, n>1.$

b) Show that $n! = O(n^n)$
but $n^n \neq O(n!)$

c) Prove that the complete graph K_5 is not planar.

(6+6+6)