C3-R4: MATHEMATICAL METHODS FOR COMPUTING

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) If a box contains 'x' good VLSI chips and 'y' defective chips. Now 'z' chips are selected at random. Find the probability that at least one chip is defective. Assume z < x.

b) The mean arrival rate at a milk booth is one customer every 4 minutes and the mean service time is 2½ minutes. If the waiting cost is Rs.5 per unit per minute and the cost of servicing one unit is Rs. 4, then find the minimum cost service rate.

c) Let $f(x) = x,\ 0 < x < 2\pi$

Find the Fourier coefficients a_n and b_n, n = 1, 2, 3, …, for f(x).

d) Suppose that people immigrate into a territory at a Poisson rate $\lambda=1$ per day.

i) What is the expected time until the 10^{th} immigrant arrives?

ii) What is the probability that the elapsed time between the 10^{th} and 11^{th} arrival exceeds two days?

e) Convert the following linear programming problem to *standard form*:

$$\begin{aligned} \text{maximize} \quad & 2x_1 + x_2 \\ \text{subject to} \quad & 0 \le x_1 \le 2 \\ & x_1 + x_2 \le 3 \\ & x_1 + 2x_2 \le 5 \\ & x_2 \ge 0 \end{aligned}$$

f) Let $f(t) = (\frac{1}{2})(e^{at} + e^{-at})$. Find the Laplace transform of f.

g) Find E[X] where X is the outcome when we roll a fair die.

(7x4)

2.

a) The capacity of a wireless communication channel is 20000 bits per second (bps). This channel is used to transmit 8-bit characters. The application calls for traffic from many devices to be sent on the channel with a total volume of 120000 characters per minute. Calculate the average number of characters waiting in the channel and the average transmission time per character (including waiting time).

b) Let X denote a random variable that takes on any of the values -1, 0, 1 with respective probabilities

P{X= -1} = 0.2 , P{X=0}=0.5 , P{X=1}=0.3

Compute $E[X^2]$.

(9+9)

3.

a) Find Inverse Laplace Transform of $(s+7) / (s^2 - 3s - 10)$.

b) Find the Fourier coefficients of the periodic function $f(x)$ and the Fourier series where

$$f(x) = -k \quad \text{if} \quad -\pi < x < 0$$
$$f(x) = k \quad \text{if} \quad 0 < x < \pi$$
$$\text{and} \quad f(x) = f(x+2\pi)$$

(9+9)

4.

a) Consider the following linear program

maximize $z=2x_1 + 5x_2$

subject to $x_1 \le 4$

$x_2 \le 6$

$x_1 + x_2 \le 8$

$x_1 , x_2 \ge 0$

Solve it using simplex method.

b) Consider the linear program

minimize $z=4x_1 + 3x_2$

subject to $5x_1 + x_2 \ge 11$

$2x_1 + x_2 \ge 8$

$x_1 + 2x_2 \ge 7$

$x_1 , x_2 \ge 0$

Write the corresponding dual problem, and find the solution to the dual.

(9+9)

5.

a) Let X be a gamma random variable with parameters α and λ. Calculate E[X] and Var[X].

b) The joint density of X and Y, two continuous random variables is given by

$f(x,y) = e^{-(x+y)}$ for $0 < x < \infty$ and $0 < y < \infty$

$f(x,y) = 0$ otherwise

Find the density function of Z=X/Y

(9+9)

6.

a) Let X be a random variable that takes on 3 possible values with respective probabilities 0.35, 0.75, and 0.5. Find the entropy H(X) for the random variable X.

b) An investor has 20 thousand dollars to invest among 4 possible investments. Each investment must be in units of thousand dollars. If the total 20 thousand is to be invested, how many different investment strategies are possible?

c) Laplace transform of *f(t) and g(t)* is *F(s)* and *G(s).* What is the Laplace transform of $af(t)+b^2g(t)$.

(6+6+6)

7.

a) If X and Y are independent Poisson random variables with respective parameters λ_1 and λ_2 , compute the distribution of X+Y.

b) A subset of nodes is called an Independent set if no two of them are adjacent. Suppose you are given a simple graph G which is a path i.e. v1, v2, v3…vN. For each v_i there is a weight w_i. The goal is to find an independent set in the path G whose total weight is maximum. Write the recurrence relation for solving the problem. Give an idea of how the problem can be solved using an algorithm that does not use recursion.

(9+9)