B0-R4: BASIC MATHEMATICS

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Find Re(z) and Im(z) where $z=\left(\frac{i}{3-i}\right)\left(\frac{1}{2+3i}\right)$.

b) If $A=\begin{bmatrix}2 & 3\\ 4 & 7\end{bmatrix}$, then find a matrix B such that 2A + 3B = A^2.

c) Find $\lim_{x\to 0}\frac{\sqrt{2+3x}-\sqrt{2-5x}}{4x}$.

d) If $x^y = e^{x-y}$, then show that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$.

e) Find a curve in the xy – plane that passes through the point (0, 3) and whose tangent line at a point (x, y) has slope $\frac{2x}{y^2}$.

f) Show that the series $\left[\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+...\right]$ is convergent.

g) Show that the vectors (2, -3, 1) and (1, 2, 4) are orthogonal.

(7x4)

2.

a) Find the rank of the matrix $A=\begin{bmatrix}3 & 0 & 2 & 2\\ -6 & 42 & 24 & 54\\ 21 & -21 & 0 & -15\end{bmatrix}$.

b) Find the eigen values of the matrix $\begin{bmatrix}-5 & 2\\ 2 & -2\end{bmatrix}$.

c) Using Cramer's rule, find the solution to the following system of linear equations:

$3x+ 3y - z = 11$
$2x- y +2z = 9$
$4x + 3y + 2z = 25$.

(6+4+8)

3.

a) For what values of the constant k the function

$$f(x)=\begin{cases}\frac{x^2-3x-2}{x-1}, & x\neq 1\\ k, & x=1\end{cases}$$

is continuous at x = 1? Explain.

b) Let m and n be positive integers. If $x^m y^n = (x + y)^{m+n}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.

c) Locate (if any) the relative maxima and relative minima of the function $f(x) = 3x^{5/3} - 15x^{2/3}, x > 0$.

(5+5+8)

4.

a) Verify the hypotheses of the Mean Value Theorem on the interval [3, 4] for the function $f(x) = x + (1/x)$ and find the value of c in [3, 4] which satisfies the conclusion of the theorem.

b) Find the slope of the tangent line to the unit circle

$x = \cos t, \quad y = \sin t, \quad 0 \le t \le 2\pi\}$

at the point where $t = \pi/6$.

c) Find an equation of the parabola that is symmetric about the y – axis has its vertex at the origin, and passes through the point (5, 2).

(6+6+6)

5.

a) Evaluate $\int \frac{sec^2(\sqrt{x})}{\sqrt{x}} dx$.

b) Find the area of the region enclosed by $x = y^2$ and $y = (x - 2)$.

c) Solve the differential equations $\frac{dy}{dx} - y = e^{2x}$.

(6+6+6)

6.

a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n!10^n}{3^n}$.

b) Consider two lines L_1 and L_2 in 3-dimension whose parametric equations are given as follows:

L_1: $x = 1 + 4t$, $y= 5 - 4t$, $z = -1 + 5t$

L_2: $x = 2 + 8t$, $y = 4 - 3t$, $z = 5 + t$

where $t \in$ IR. Are the two lines parallel? Explain.

c) Find the equation of a plane passing through the points (1, 1, 0), (0, 1, 1), (1, 0, 1).

(6+6+6)

7.

a) Evaluate $\int \left(\frac{1}{log\, x} - \frac{1}{(log\, x)^2} \right) dx$.

b) Find a vector $\vec{n}$ which is normal to the vectors $\vec{x} = 4\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{y} = 8\hat{i} - 3\hat{j} + \hat{k}$.

c) Find the first four terms of the Maclaurin's series at a=0 for $f(x) = \frac{1}{1-x}$.

(6+6+6)