B4.1-R4: COMPUTER BASED STATISTICAL & NUMERICAL METHODS

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) A problem in mechanics is given to three students A, B and C whose chances of solving it are 1/2, 1/3 and 1/4 respectively. What is the probability that the problem is solved?

b) Find the value of λ for which the equations

$$2x + y + 2z = 0$$
$$x + y + 3z = 0$$
$$4x + y + \lambda z = 0$$

Have a non zero solution.

c) X is a continuous random variate with pdf

$$f(x) = \begin{cases} Axe^{-x^2} & ; \quad x \geq 0 \\ 0 & ; \quad x < 0 \end{cases}$$

Determine the constant and find the cdf.

d) Three approximate values of the number 1/3 are given as 0.30, 0.33 and 0.34. Which of these three is best approximation?

e) X is a binomial variate with parameters n = 10 and p = 1/3. Calculate E[X].

f) If $X_1, X_2, \ldots, X_n$ constitute a random sample from a population with the mean μ and the variance σ^2, use the method of moments to find estimators for μ and σ^2.

g) Define the term absolute error. Given that

$a = 10.00 \pm 0.05$
$b = 0.0356 \pm 0.0002$
$c = 15300 \pm 100$
$d = 62000 \pm 500$,

find the maximum value of the absolute error in

i) $a + b + c + d$
ii) $a + 5c - d$
iii) c^3

(7x4)

2.

a) Using Gauss-Seidel method, solve the following system of equations:

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$
$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$
$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$
$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

You may start from an approximation

$\underset{\sim}{X}^{(0)} = (0.3000, 1.5600, 2.8860, -0.1368)^T$

b) Use the Newton-Raphson method to find a root of the equation $x^3 - 2x - 5 = 0$.

(10+8)

3.

a) The joint pdf of the bivariate random variable (X, Y) is

$$f(x,y)=\begin{cases}\frac{1}{8}(x+y) & ; \quad 0 \le x, y \le 2 \\ 0 & ; \quad elsewhere\end{cases}$$

Are X and Y independent random variables? Compute E[X] and E[Y].

b) Random variable Z has zero mean and unit variance. Given Z = X – 1 and Y = Z + 1, Compute E[XY] and correlation coefficient P_{XY}.

(9+9)

4.

a) If the probability of hitting a target is 1/5 and ten shots are fired, then

i) What is the probability that the target will be hit at least twice?

ii) What is the conditional probability of the target being hit atleast twice given that atleast one hit was scored?

b) The lifetime T of light bulb is a random variable with exponential pdf

$f_T(t) = 0.001 \exp[-0.001t],\ t \ge 0$

i) Find the probability that the bulb will last 2000 hours.

ii) Obtain mean life time of the bulb.

(10+8)

5.

a) The hardness of steel, in standardized units, varies with the percentage of tungsten added. The results of y readings are

% tungsten	0.20	0.40	0.60	0.80	1.00	1.20
Hardness y	0.13	0.24	0.51	0.72	1.05	1.34

i) Compute $\bar{x}$ and $\bar{y}$.

ii) Find the best estimates for regression curve of the form y = a + bx.

b) Suppose p(x, y) is the joint probability mass function of X and Y such that

p(1, 1) = 0.5, p(1, 2) = 0.1,

p(2, 1) = 0.1, p(2, 2) = 0.3

Compute $p_{X|Y}(1|1) = P\{X = 1 \mid Y = 1\}$

$p_{X|Y}(2|1) = P\{X = 2 \mid Y = 1\}$

(10+8)

6.

a) If X_1 and X_2 are independent Poisson random variates with respective means 1 and 2 respectively, obtain $P(X_1 + X_2 = 2)$.

b) If X is normally distributed with mean 1 and variance 4, find P(2 < X < 3).

c) A coin, having probability p of landing heads, is flipped until the head appears for the first time. Let N denote the number of flips required. Calculate E[N].

(6+6+6)

7.

a) The population of a town in decennial census were as under. Using Newton's forward difference interpolation formula, estimate the population for the year 1955.

Year	**Population (in thousands)**
1921	46
1931	66
1941	81
1951	93
1961	101

b) Given x successes in n trials, find the maximum likelihood estimate of the parameter θ of the corresponding binomial distribution.

c) If X has the binomial distribution with the parameters n and θ, show that the sample proportion $\frac{X}{n}$ is a unbiased estimator of θ.

(6+6+6)