B3.2-R4: DISCRETE STRUCTURE

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Let N be the set of natural numbers and f, g, h are functions from N to N defined by

$$f(n) = n + 1$$

$$g(n) = 2n$$

Determine $f \circ g$ and $g \circ f$.

b) Minimize the Boolean expression $F = \overline{A}\,C + \overline{A}\,B + A\overline{B}C + BC$.

c) By using the pigeonhole principle, show that if any five numbers from 1 to 8 are chosen, then two of them will add to 9.

d) Find the greatest common divisor of 117 and 45.

e) Construct truth table to determine whether $\sim(A \rightarrow B) \vee (\sim A \vee (A \wedge B))$ is a tautology.

f) Find a grammar that generates the language $L = \{0^n 1^n : n \geq 0\}$.

g) Consider the following specification of a graph G

Vertex set of G = {1,2,3,4}

Edge set of G = {(1,2), (1,3), (3,3), (3,4), (4,1)}

i) Draw an undirected graph

ii) Write its adjacency matrix.

(7x4)

2.

a) If $f = (2\ \ 3)$ and $g = (4\ \ 5)$ be two permutations on five symbols $\{1,2,3,4,5\}$, then prove that $f \circ g = g \circ f$.

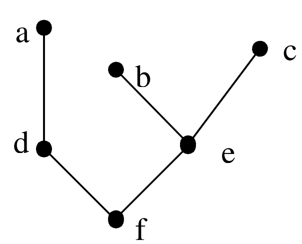
b) Prove that a connected planar graph with *n* vertices and *e* edges has $e - n + 2$ regions.

c) Use the mathematical induction to prove that $n(n+1)(2n+1)$ is divisible by 6.

(6+6+6)

3.

a) Consider the poset whose Hasse diagram is given below:



i) List all the maximal elements of the poset.

ii) List all the minimal elements of the poset.

iii) Draw a new Hasse diagram by adding a single element z to the top of the above Hasse diagram and join it with vertices a, b, c by lines. Verify that the new poset is a Lattice.

b) Find the partitions corresponding to 0-equivalence, 1-equivalence and 2-equivalence (π_0, π_1 and π_2) by minimizing the finite state machine whose stable table is given below:

	Input		
State	**0**	**1**	**Output**
S_0	S_3	S_1	1
S_1	S_4	S_1	0
S_2	S_3	S_0	1
S_3	S_2	S_3	0
S_4	S_1	S_0	1

(8+10)

4.

a) Using the Karnaugh map, find a minimal form of Boolean function

$$f(x, y, z) = xyz + xyz' + x'yz' + x'y'z' + x'y'z .$$

b) Sort the following list in ascending order using merge sort algorithms. Describe the steps of the algorithm in detail.

32, 51, 27, 85, 66, 23, 13, 57

(9+9)

5.

a) In how many ways can a committee of 5 persons can be formed from 6 men and 4 women so as to include at least 2 women?

b) Solve the recurrence relation

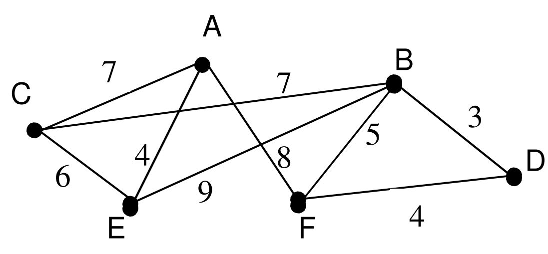
$$a_{r+2} - 3a_{r+1} + 2a_r = 0,\ a_0 = 2,\ a_1 = 3.$$

c) Prove the validity of the following argument using truth tables:
"If it rains then it will be cold.
If it is cold then I shall stay at home.
Since it rains therefore, I shall stay at home."

(6+6+6)

6.

a) Use the Kruskal's Algorithm to find a minimum spanning tree in the following graph:



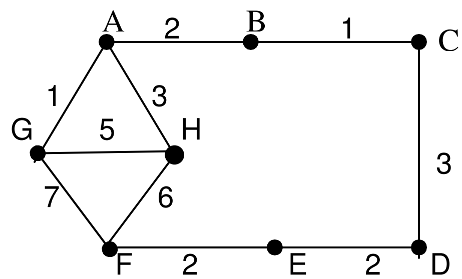
b) Let G = $\{$ n : $1 \le n \le 11$ and the greatest common divisor between n and 11 is 1 $\}$, and $\odot_{11}$ be a multiplication modulo 11 operator. Prove that (G, $\odot_{11}$) is a group.

(9+9)

7.

a) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3 and 7.

b) Solve the traveling salesman problem for the graph given below by finding the minimum Hamiltonian Circuit in it.



c) Let R be a relation on set $A = \{1,2,3,4\}$ defined by

$R = \{(1,1),\ (2,2),\ (3,3),\ (4,4),\ (4,3),\ (4,2),\ (4,1),\ (3,2),\ (3,1)\}$

Find the matrix and directed graph of relation R.

(6+6+6)