B4.1-R4: COMPUTER BASED STATISTICAL & NUMERICAL METHODS

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) The probability that a student is accepted to a prestigeous college is 0.3. If 5 students from the same school apply, what is the probability that at most 2 are accepted?

b) For the bivariate pdf

$$f(x,y)=\begin{cases} xe^{-x(y+1)}, & x\ge 0, y\ge 0 \\ 0 & \text{otherwise} \end{cases}$$

compute the conditional density $f_{Y|X}(y|x)$.

c) A random variable has probability density of the form

$$f(x)=\begin{cases} cx^2 & 0\le x\le 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the constant c. Also obtain $P(x \le a)$ for $0 \le a \le 1$.

d) Which is a superior method: the direct Gauss Elimination method or the Iterative Gauss Seidel method? Justify your answer.

e) X is a geometric random variable with probability p. Write its probability mass function. Hence find E[X].

f) X_1 and X_2 are independent random variables with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively. Find $Var(2X_1 + 3X_2)$.

g) Obtain divided difference table for the function values:

x:	3	1	5	6
f(x):	1	-3	2	4

(7x4)

2.

a) A family has two children. What is the conditional probability that both are boys given that at least one of them is a boy?

b) Let X be a gamma random variable with parameters α and λ. Calculate
i) E[X] and (ii) Var(X)

c) A random variate X is normally distributed with mean 50 and variance 25. Determine $P(|X - 50| < 8)$.

(6+6+6)

3.

a) For random variables X and Y, the joint probability density function is given by

$$f_{X,Y}(x,y)=\begin{cases} \dfrac{1+xy}{4} & |x|\le 1, |y|\le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal densities $f_X(x)$ and $f_Y(y)$. Also obtain $f_{Y|X}(y/x)$. Are X and Y independent?

b) Consider the random variables X and Y with the joint probability mass function as presented in the following table

Y \ X	0	1	2
0	0.25	0.1	0.15
1	0.14	0.35	0.01

Calculate marginal probabilities and the conditional PMF of Y given X=1.

(10+8)

4.

a) A survey of 800 families with four children each revealed the following distribution:

No of boys	0	1	2	3	4
No of girls	4	3	2	1	0
No of families	32	178	290	236	64

Is this result consistent with the hypothesis that male and female births are equally probable?

b) The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches? Test at 5% significance level. Given that for 9 degrees of freedom $P(t > 1.83) = 0.05$.

(10+8)

5.

a) Consider the system

$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & 6 & -2 \\ 4 & -3 & 8 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 5 \end{bmatrix}$$

Apply the Gauss-Seidel iteration with $X^{(0)} = (0,0,0)^T$.

b) The following data gives the melting point of an alloy of lead and zinc, where y is the temperature in ^oC and x is the % of lead in the alloy:

x:	40	50	60	70	80	90
y:	184	204	226	250	276	304

Using Newton interpolation formula, find the melting point of the alloy containing 84% of lead.

c) A river is 80 meter wide. The depth y in meter at a distance x meters from one bank is given by:

x:	0	10	20	30	40	50	60	70	80
y:	0	4	7	9	12	15	14	8	3

Using Simpson's rule, calculate the area of cross section of the river.

(6+6+6)

6.

a) Suppose the observations on X and Y are given as:

X:	59	65	45	52	60	62	70	55	45	49
Y:	75	70	55	65	60	69	80	65	59	61

where X and Y represent marks in Mathematics and Economics respectively of 10 students. Compute the least square regression equations of Y on X and X on Y.

b) Two variables X and Y are connected by the equation ax+by+c=0. Show that the correlation between them is -1 if the signs of a and b are alike and +1 if they are different.

(9+9)

7.

a) Find the maximum likelihood estimator for p when

$$f(x, p) = p^{x}(1 - p)^{1-x} \quad \text{for } x = 0, 1$$

b) The mean weight loss of n=16 grinding balls after a certain length of time in mill slurry is 3.42 gm with a standard deviation of 0.68 gm. Construct a 99% confidence interval for the true mean weight loss of such grinding balls under the stated conditions.

c) Show that the sample variance

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})^2$$

is an unbiased estimator of population variance σ^2.

(6+6+6)