B3.2-R4: DISCRETE STRUCTURE

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

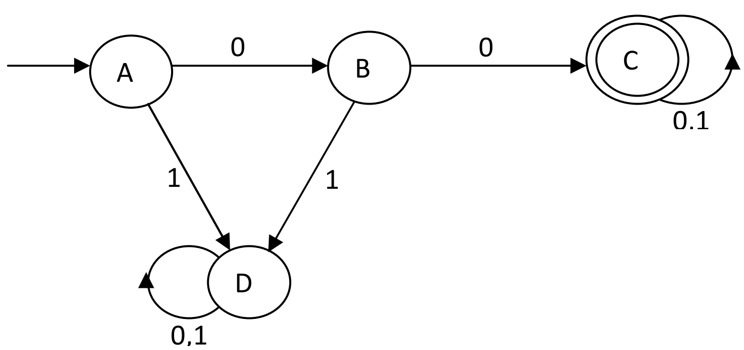
1.

a) Let U be the set of real numbers, $A = (x \mid x$ is a solution of $x^2 - 4 = 0)$ and $B = \{-1, 4\}$. Compute $\overline{A \cup B}$ and $\overline{A \cap B}$?

b) A bank password consists of two letters of the English alphabet followed by two digits. How many different passwords are there?

c) Why can there not exist a graph whose degree sequence is 5, 4, 4, 3, 2, 2, 1?

d) What kind of string does the following automaton reject?



e) Let $A = \{1,2,3,4,5,6\}$ and $\leq$ denote the partial order of divisibility. Draw the Hasse diagram for the poset $(A, \leq)$ and find all minimal, minimum, maximal and maximum elements.

f) Find the generating function of the sequence $0,1,2,3,4 \ldots\ldots$?

g) Let G be the set of all nonzero real numbers and let

$$a * b = \left(\frac{ab}{2}\right).$$

Find, if exist, the identity element and the inverse of every element in G.

(7x4)

2.

a) Determine the validity of each of the following argument.

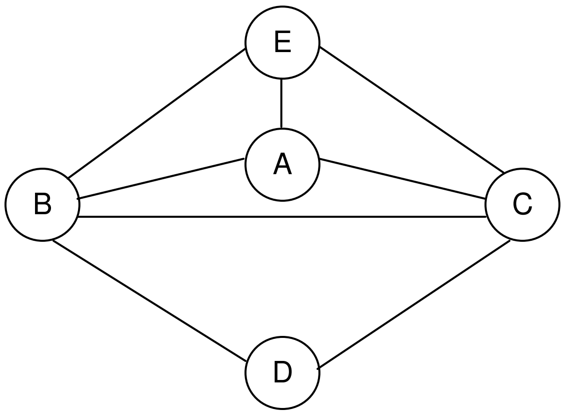
If I like mathematics, then I will study.

Either I don't study or I pass mathematics

<u>If I don't graduate, then I didn't pass mathematics</u>

If I graduate, then I studied.

b) Consider the graph shown below:



i) Is it a Hamiltonian graph? Justify your answer.
ii) Is it Eulerian graph? Justify your answer.
iii) Is there an Eulerian trail? Justify your answer.

c) Using Euclidean Algorithm (EA) find GCD (190, 34).

(8+6+4)

3.

a) A tree has n vertices of degree 2, $2n$ vertices of degree 3 and $3n$ vertices of degree 1. Determine the number of vertices and edges in the tree?

b) For any natural number n, let $f(n)=17n^4+8n^3+5n^2+6n+1$ and $g(n)=n^4$. Show that $f=O(g)$.

c) The Fibonacci sequence can be obtained using the recurrence relation

$$f_n = f_{n-1} + f_{n-2}$$

with initial conditions $f_1=1$ and $f_2=1$. Find the solution for f_n?

(6+6+6)

4.

a) Design an FA (Finite Automata) that accepts all binary strings that begin and end with the same symbol.

b) In a survey of 260 students, the following data were obtained:

64 had taken a mathematics course,
94 had taken a computer science course,
58 had taken a business course,
28 had taken a mathematics and a business course,
26 had taken a mathematics and a computer science course
22 had taken a computer science and a business course, and
14 had taken all three types of course.

i) Of the students surveyed, how many had not taken any of the three courses?
ii) Of the students surveyed, how many had taken only computer science?

(9+9)

5.

a) Let $(S, \circ)$ be symmetry group of an equilateral triangle and let

$$H = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \right\}$$

be a subgroup of $(S, \circ)$. Find $(H \circ p_3)$ and $p_3 \circ H$, where $p_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \in S$.

b) Describe an algorithm that, upon input of $n+1$ integers $a_0, a_1, a_2 \cdots\cdots, a_n$ and an integer x, outputs the integer $a_0 + a_1 x + a_2 x^2 + \cdots\cdots\cdots\cdots + a_n x^n$.

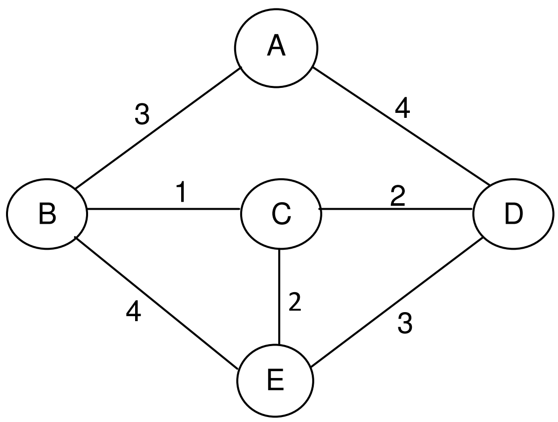
c) Sort the following list in ascending order using merge sort algorithm. Describe the steps of the algorithm in detail.

2 9 1 4 6 5 3

(6+6+6)

6.

a) Determine a minimum spanning tree for the connected network G shown below using Kruskal algorithm:



b) Show that the relation R={(a,b):a – b is an even integer and a, b ∈ I} is an equivalence relation?

c) Explain what language this CFG (Context-free Grammar) generates (with start symbol S):

$$S \rightarrow AB$$
$$A \rightarrow 0A1 \mid \varepsilon$$
$$B \rightarrow 1B0 \mid \varepsilon$$

Sketch a proof that our answer is corrects.

(6+6+6)

7.

a) Let G be a connected planar graph such that every region of G has at least five edges on its boundary. Prove that $3E \leq 5V - 10$, where E is the number of edges and V is the number of vertices in graph G.

b) Use mathematical induction to prove that 5^n-1 is divisible by 4 for every natural number n.

(10+8)