B4.1-R3: COMPUTER BASED STATISTICAL & NUMERICAL TECHNIQUES

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Five persons in a group of 20 are graduate. If 3 persons are picked out of 20 at random, (i) compute the probability that all are graduate, (ii) find the probability of atleast one being graduate.

b) Suppose we are given a discrete random variable with P(X=0)=P(X=2)=0.25 and P(X=1)=0.5. Obtain E[X] and V[X].

c) Using Simpson's rule, evaluate $\int_{-3}^{3} x^4 dx$ by taking seven equidistance ordinates.

d) A die is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Test the null hypothesis H_O that the die is an unbiased one.

e) Construct a forward difference table from the following data:

x	0	1	2	3
y(x)	1	0	1	10

f) X is normally distributed and the mean of X is 12 and standard deviation is 4. Find (i) $P(X \geq 20)$, (ii) $P(0 \leq X \leq 12)$.

g) If $X_1, X_2, \ldots, X_n$ constitute a random sample from an infinite population with mean μ and the variance σ^2, show that $\overline{X}$ is an unbiased estimator for population mean μ.

(7x4)

2.

a) Find the Lagrange interpolating polynomial of degree 2 approximating the function $y = \ell n\ x$ defined by the following table of values. Hence determine the value of $\ell n\ 2.7$.

x	$y = \ell n\ x$
2	0.69315
2.5	0.91629
3.0	1.09861

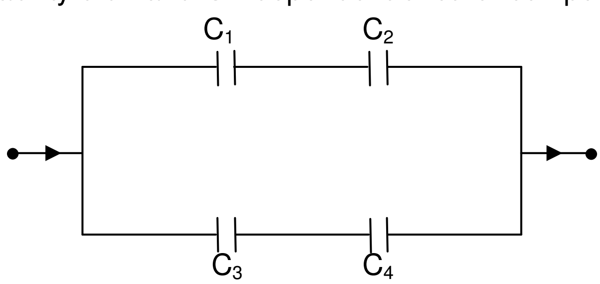
b) Solve the following system of linear equations by Gauss elimination method:

$$x_1 + x_2 + x_3 = 6$$
$$3x_1 + 3x_2 + 4x_3 = 20$$
$$2x_1 + x_2 + 3x_3 = 13.$$

(9+9)

3.

a) Calculate the probability that the system works if each component is operable with probability 0.92 and is independent of other components.



b) Random variate X follows normal distribution with mean 0 and variance 1, i.e. $X \sim N(0, 1)$. Given $Y = 2X + 4$, find
(i) $E[Y]$, (ii) Var Y (iii) $E[X^3]$.

c) A new computer virus attacks a folder consisting of 200 files. Each file gets damaged with probability 0.2 independently of other files. Using normal approximation to Binomial distribution, find the probability that fewer than 50 files get damaged.

(6+6+6)

4.

a) The time in minutes, it takes to reboot a certain system is a continuous random variable with density

$$f(x) = \begin{cases} C(10 - x^2) & , \quad 0 \le x \le 10 \\ 0 & , \quad \text{otherwise} \end{cases}$$

i) Compute C.
ii) Obtain the probability that it takes between 1 and 2 minutes to reboot.

b) A program consists of two modules. The number of errors X in the first module and the number of errors Y in the second module have the joint distribution:

$P(0, 0) = P(0, 1) = P(1, 0) = 0.20$
$P(1, 1) = P(1, 2) = P(1, 3) = 0.10$
$P(0, 2) = P(0, 3) = 0.05$

Find
i) the marginal distribution of X and Y.
ii) the probability of number errors in the first module.
iii) the probability distribution of the total number of errors in the program.
iv) if the errors in the two modules occur independently?

(6+12)

5.

a) The joint probability distribution function of X and Y given below

Y \ X	-1	+1
0	$\frac{1}{8}$	$\frac{3}{8}$
1	$\frac{2}{8}$	$\frac{2}{8}$

Find the correlation coefficient between X and Y.

b) Obtain the equations of two lines of regression for the following data. Also find the estimate of X for Y=70.

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

(8+10)

6.

a) A survey of 800 families with four children each revealed the following distribution

No. of boys	0	1	2	3	4
No. of girls	4	3	2	1	0
No. of families	32	178	290	236	64

Is this result consistent with the hypothesis that male and female births are equally probable? Use χ^2_4 (0.05)=9.488.

b) In a random sample of 400 students of the university teaching departments, it was found that 300 students failed in the examination. In another random sample of 500 students of the affiliated colleges, the number of failures in the same examination was found to be 300. Find out whether the proportion of failures in the university teaching departments is significantly greater than the proportion of failures in the university teaching departments and affiliated colleges taken together.

(9+9)

7.

a) X_1, X_2 and X_3 is a random sample of size 3 from a population with mean value μ and variance σ^2. T_1 and T_2 are the estimators to estimate the mean value μ, where $T_1 = X_1 + X_2 - X_3$, $T_2 = (\lambda X_1 + X_2 + X_3)/3$

i) Is T_1 an unbiased estimator for μ?

ii) Find the value of λ such that T_2 is an unbiased estimator for μ.

iii) Which is the best estimator out of T_1 and T_2?

b) A machinist is making engine parts with axle diameters of 0.700 inch. A random sample of 10 parts shows a mean diameter 0.742 inch with a standard deviation of 0.040 inch. Compute the statistic you would use to test whether the work is meeting the specification.

c) X and Y are independent Poisson variates with mean λ_1 and λ_2 respectively. Find

i) $P(X + Y = 1)$

ii) $P(X = Y)$

(6+6+6)