B4.1-R4: COMPUTER BASED STATISTICAL & NUMERICAL METHODS

NOTE:
1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours                        Total Marks: 100

1. a) A fair die is rolled three times and the scores added. What is the probability that sum of scores is 6?
    b) It is known that all items produced by a certain machine will be defective with probability 0.1, independently of each other. Find the probability that in a sample of three items, at most one will be defective.
    c) If \( u = 3v^2 - 6v \), find the percentage error in \( u \) at \( v = 1 \) if the error in \( v \) is 0.05.
    d) Find from the following table, the area bounded by the curve and the x-axis from \( x = 7.47 \) to \( x = 7.52 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>7.47</th>
<th>7.48</th>
<th>7.49</th>
<th>7.50</th>
<th>7.51</th>
<th>7.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>1.93</td>
<td>1.95</td>
<td>1.98</td>
<td>2.01</td>
<td>2.03</td>
<td>2.06</td>
</tr>
</tbody>
</table>

e) If \( X \) and \( Y \) are independent variables, find \( \text{Cov}(X, Y) \).

f) If \( X \) is a normal variate with mean \( \mu \) and variance \( \sigma^2 \), find variance of \( Y = 2X + 1 \).

g) The two lines of regression are given as
   \[
   \begin{align*}
   X + 2Y - 5 &= 0 \\
   2X + 3Y &= 8 
   \end{align*}
   \]
   Compute the mean value of \( X \) and \( Y \).

(7x4)

2. a) Use bisection method to find the positive root of 30 correct to two decimal place.
    b) Perform four iterations of the Newton-Raphson method to find the smallest positive root of the equation
       \[ f(x) = x^3 - 5x + 1 = 0 \]
       The smallest positive root lies in the interval \((0, 1)\). Take the initial approximation as \( x_0 = 0.5 \).
    c) Given \( f(2) = 4 \), \( f(2.5) = 5.5 \), find the linear interpolating polynomial using Lagrange interpolation.

(6+6+6)

3. a) Evaluate \[ \int_0^1 \frac{dx}{1 + x} \]
    by dividing the interval of integration into eight equal parts. Hence find \( \log_e 2 \) approximately.
    b) Use Gauss elimination to solve
       \[
       \begin{align*}
       2x + y + z &= 10 \\
       3x + 2y + 3z &= 18 \\
       x + 4y + 9z &= 16 
       \end{align*}
       \]

(8+10)
4.  
   a) Suppose that X is a continuous random variable whose probability density function (pdf) is  
   given by  
   \[ f(x) = \begin{cases} 
   c(4x-2x^2) & 0 < x < 2 \\
   0 & \text{otherwise}
   \end{cases} \]  
   i) What is the value of C?  
   ii) Find P(X > 1).  
   b) The joint pdf of the bivariate random variable \((X, Y)\) is  
   \[ f(x, y) = \begin{cases} 
   \frac{1}{8}(x+y) & 0 \leq x, y \leq 2 \\
   0 & \text{otherwise}
   \end{cases} \]  
   Find marginal pdf's of X and Y. Are X and Y independent?  

5.  
   a) Suppose that the number of typographical errors on a single page of a book has Poisson  
   distribution with parameter \(\lambda = \frac{1}{2}\). Calculate the probability that there is at least  
   one error on a page.  
   b) Compute Var X where X represents the outcome when we roll a fair dice.  
   c) X is a Binomial variate with parameters \(n\) and \(p\). The mean and variance of X are 3 and 2.1,  
   respectively. Find \(n\) and \(p\).  

6.  
   a) The yield of chemical A in litres of a fermentation process is related to the temperature  
   during fermentation. The results of a sequence of experiments are as follows:  
   \[
   \begin{array}{|c|c|c|c|c|c|c|}
   \hline
   X (\degree C) & 35 & 40 & 45 & 50 & 55 & 60 \\
   \hline
   Y (litres) & 20.2 & 23.1 & 23.2 & 23.6 & 25.8 & 26.3 \\
   \hline
   \end{array}
   \]  
   i) Find the regression of \(Y\) and \(X\).  
   ii) Estimate the average yield if the fermentation temperature is 48 \degree C.  
   b) The number of defects found on a roll of carpet has a Poisson distribution with parameter \(\lambda\).  
   If four rolls of carpet are inspected and found to have 12, 4, 9 and 15 defects, respectively,  
   find maximum likelihood for \(\lambda\).  

7.  
   a) The following data come from a normal population having standard deviation 4.  
   \[105, 108, 112, 121, 100, 105, 99, 107, 112, 122, 118, 105\]  
   Use them to test the null hypothesis that the population mean is less than or equal to 100 at  
   i) 5 percent level of significance.  
   ii) 1 percent level of significance.  
   iii) What is the \(p\) value?  
   b) The manufacturer of a new fibre glass tyre claims that the average life of a set of its tyres is  
   at least 50,000 km. To verify this claim, a sample of 8 sets of tyres was chosen and the tyres  
   subsequently were tested by a consumer agency. If the resulting values of the sample mean  
   and sample variance were, respectively 47.2 and 3.1 (in 1,000 kms.), test the  
   manufacturer’s claim at 5% level of significance.