

C0-R4.B1: ELEMENTS OF MATHEMATICAL SCIENCES

NOTE :

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1. (a) Evaluate the determinant of matrix A

$$A = \begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}$$

- (b) Change to Cartesian coordinates the equation
- (i) $r = a \sin \theta$ and
 - (ii) $r^{\frac{1}{2}} = a^{\frac{1}{2}} \cos \frac{\theta}{2}$
- (c) Find the asymptotes of the graph of $(x) = \frac{x^2-3}{2x-4}$.
- (d) Evaluate the integral (i) $\int \sqrt{4t-1} dt$ (ii) $\int x^2 \sin(x^3) dx$.
- (e) Determine whether the series $\frac{3}{4} + \frac{5}{9} + \frac{7}{16} + \frac{9}{25} + \dots$ converge or diverge.
- (f) Let X have the probability density function $f(x) = 0.75(1-x^2), -1 \leq x \leq 1$ and zero elsewhere. Find the probabilities.
- (i) $P\left(-\frac{1}{2} \leq X \leq \frac{1}{2}\right)$,
 - (ii) $P\left(\frac{1}{4} \leq X \leq 2\right)$
- (g) Compute the probability of obtaining at least two "Six" in rolling a fair die 4 times.

(7×4)

2. (a) Find the eigenvalues and eigenvectors of matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

- (b) Find (i) $\lim_{x \rightarrow +\infty} (\sqrt{x^6+5} - x^3)$ (ii) $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+2}}{3x-6}$

(9+9)

3. (a) (i) Find the equation to the tangents to the circle $x^2 + y^2 - 6x + 4y = 12$ which are parallel to the straight line $4x + 3y + 5 = 0$.
- (ii) Find the vertex, axis, focus, and latus rectum of the parabola $4y^2 + 12x - 20y + 67 = 0$.
- (b) Solve the system of equations using Gauss elimination

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned}$$

(9+9)

4. (a) (i) Let $u = (2, -1, 3)$ and $a = (4, -1, 2)$. Find the vector component of u along a and vector component of u orthogonal to a .
(ii) Find the vector product $v = a \times b$, where $a = (1, 1, 0)$ and $b = (3, 0, 0)$
(b) Evaluate
(i) $\int_{\pi/2}^{\pi/4} (x^2 + 2x - 3)^2 (x + 1) dx$
(ii) $\int_{\pi/4}^{\pi/2} \cot \theta \csc^2 \theta d\theta$

(9+9)

5. (a) Investigate the convergence of series (i) $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$ (ii) $a_n = \begin{cases} \frac{n}{2^n}, n \text{ odd} \\ \frac{1}{2^n}, n \text{ even} \end{cases}$
(b) Find the Taylor series and Taylor polynomials generated by $f(x) = \cos x$ at $x = 0$.

(9+9)

6. (a) (i) A box contains 10 screws, three of which are defective. Two screws are drawn at random. Find the probability that neither of the two screws is defective.
(ii) If the probability of producing a defective screw is $p = 0.01$, what is the probability that a lot of 100 screws will contain more than 2 defectives?
(b) In a production of iron rods let the diameter X be normally distributed with mean 2 in. and standard deviation 0.008 in.
(i) What percentage of defectives can we expect if we set the tolerance limits at 2 ± 0.02 in.?
(ii) How should we set the tolerance limits to allow for 4% defectives?

(9+9)

7. (a) Five independent measurements of the point of inflammation (flash point) of Diesel oil gave the values (in $^{\circ}F$) 144 147 146 142 144. Assuming normality, determine a confidence interval for the mean.
(b) Using the given sample, test that the corresponding population has a Poisson distribution. x is the number of alpha particles per 7.5-s intervals observed, and $a(x)$ is the absolute frequency number of time periods during which exactly x particles were observed. Use $\alpha = 0.05$

x	0	1	2	3	4	5	6
a	57	203	383	525	532	408	273
x	7	8	9	10	11	12	≥ 13
a	139	45	27	10	4	2	0

(9+9)