

B0-R4 : BASIC MATHEMATICS

NOTE :

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1. (a) Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n \log n}$
(b) Solve the differential equation $y = x \frac{dy}{dx} + 3 \left(y^2 + \frac{dy}{dx} \right)$
(c) Express the complex number $\frac{1}{2}(1+i)(1+i^{-8})$ in the form $a+ib$.
(d) Determine the extreme value of the function $\sin x(1 + \cos x)$.
(e) Evaluate $\lim_{x \rightarrow 1} (1-x^2)^{\frac{1}{\log(1-x)}}$.
(f) Find the area of the region enclosed by curves $x = y^2 - 2$ and $x = y$.
(g) Find the Eigen values of the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.

(7 × 4)

2. (a) Determine whether u and v are orthogonal, parallel or neither where $u = (3, 15)$ and $v = (-1, 5)$.
(b) Verify Cayley–Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and hence find A^{-1} .

(8 + 10)

3. (a) Solve the following system of linear equations $3x - 2y + 4z = 5$, $x + y + 3z = 2$, $-x + 2y - z = 1$ by Cramer's rule
(b) Solve the differential equation $y'' + 2y' + 2y = 0$, $y(0) = 1$, $y' \left(\frac{\pi}{2} \right) = 0$.

(9 + 9)

4. (a) Find all possible values of c for the function $f(x) = x^3 - x, [0, 2]$ by the mean value theorem.

(b) Find the horizontal asymptotes of the function $f(x) = \frac{x^3 - 2}{|x|^3 + 1}$.

(c) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \tan\left(\frac{1}{n}\right)$.

(6 + 6 + 6)

5. (a) Sketch the curve $r = 2 + \cos 2\theta$ and find the area that is enclosed.

(b) Find the equation of the ellipse which passes through the point $(\pm 4, 0)$ and has eccentricity $\frac{1}{3}$ with x-axis as its major axis and centre at the origin.

(c) Using Maclaurin's theorem, expand $\log\left(\frac{1-x}{1+x}\right)$ up to the term containing x^3 .

(6 + 6 + 6)

6. (a) Find the area of the surface generated by rotating the curve $y = e^x, x \in [0, 1]$ about the x-axis.

(b) Find vector equation, parametric equations and symmetric equation for the line containing the points $P = (1, 2, -3)$ and $Q(3, -2, 1)$.

(c) Evaluate $\int \sqrt[4]{x+5} dx$.

(6 + 6 + 6)

7. (a) Find the root of the equation $x^4 + i = 0$.

(b) Find the equation of foci and the equation of hyperbola with the vertices $(0, \pm 1)$ and asymptotes $y = 2x$.

(9 + 9)