1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours
Total Marks: 100

1.

\[
\begin{bmatrix}
3 & 4 & 5 & x \\
4 & 5 & 6 & y \\
5 & 6 & 7 & 8 \\
x & y & z & 0
\end{bmatrix}
\]

a) If \( \Delta = \begin{bmatrix} 3 & 4 & 5 & x \\ 4 & 5 & 6 & y \\ 5 & 6 & 7 & 8 \\ x & y & z & 0 \end{bmatrix} \), then find the value of \( \Delta \).

b) For what value of \( \lambda \), the system of equations
\[
\begin{align*}
X + y + z &= 6 \\
X + 2y + 3z &= 10 \\
X + 2y + \lambda z &= 12
\end{align*}
\]
is in consistent.

c) If \( A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix} \) and \( |A^3| = 125 \) then find the value of \( \alpha \).

d) If \( a = i + j + k \), \( b = 4i + 3j + 4k \) ad \( c = i + \alpha j + \beta k \) are linearly dependent vectors and \( |c| = \sqrt{3} \) find the values of \( \alpha \) and \( \beta \).

e) Find the Eigen values of the matrix:
\[
\begin{bmatrix} 6 & 2 & 2 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}
\]

f) Find the rank of the matrix
\[
\begin{bmatrix} 3 & 1 & -2 \\ 2 & 0 & -1 \\ 1 & 4 & 1 \end{bmatrix}
\]

g) If \( a = 3i - j + 2k \), \( b = 2i + j - k \), \( c = i - 2j + 2k \), find \( (a \times b) \times c \) and \( a \times (b \times c) \). Are both same?

(7x4)

2.

a) For how many values of \( x \) in the closed interval \([-4, -1]\), the matrix
\[
\begin{bmatrix} 3 & -1 & x & 2 \\ 3 & -1 & x & 2 \\ x & 3 & 1 & 2 \end{bmatrix}
\]
singular.

b) If \( A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \) then find the value of \( I + A + A^2 + A^3 + \ldots \ldots \infty \).

c) Let \( a = 2i + 3j - k \) and \( b = i - 2j + 3k \). Then find the value of \( \lambda \) for which the vector \( c = \lambda i + j + (2\lambda - 1)k \) is parallel to the plane containing \( a \) and \( b \).

(6+6+6)

3.

a) An unbiased die is thrown at random. What is the expectation of the number on it?

b) In a city, 5 accidents take place in a span of 25 days. Assuming that the number of accidents follows the Poisson distribution what is the probability that there will be 3 or more accidents in a day? (Given \( e^{-2} = 0.8187 \))

c) Apply Maclaurin's expansion for expanding \( \log(1+x) \) and \( \log(1-x) \) in ascending powers of \( x \) and hence deduce the expansion of \( \log\left(\frac{1+x}{1-x}\right) \).

(6+6+6)
4. Find the covariance between \( x \) and \( y \) for the following observation \((x,y)\).

\[ \begin{array}{cccccccc}
X: & 3 & 11 & 21 & 11 & 18 & 16 & 17 \\
Y: & 5 & 16 & 13 & 17 & 18 & 11 & 12 & 9 \\
\end{array} \]

b) Find the limit of the following:

\[ \lim_{n \to \infty} \frac{7n^3 - 8n^2 + 10n - 7}{8n^3 - 9n^2 + 5} \]

c) Find the extreme values of the function \( x^3 e^{-x} \).

(6+6+6)

5. a) Integrate \( \sqrt{1+\sin 2x} \) with respect to \( x \).

b) Evaluate \( I = \int_0^{\pi/2} \log \sin x \, dx \).

c) State the values of \( a \) and \( b \) if the equation

\[ a x^2 + 2b xy - 2y^2 + 8x + 12y + 6 = 0 \]

represents a circle. Substituting the values of these \( a \) and \( b \) in the equation, find the center and radius of the circle

(6+6+6)

6. a) Integrate \( \frac{x^3}{(x^2 + 1)^3} \) with respect to \( x \).

b) Consider the function defined as follows:

\[ f(x) = \begin{cases} 
\frac{x^2 - 4}{x - 2} & \text{for } x < 2 \\
4 & \text{for } x = 2 \\
\end{cases} \]

\[ f(x) = 2, \text{ for } x > 2 \]

Discuss the continuity at \( x = 2 \).

c) If \( C_0, C_1, C_2, \ldots, C_n \) denote the coefficients of the expansion of \((1+x)^n\), prove that

i) \( C_1 + 2 C_2 + 3 C_3 + \ldots + n C_n = n \cdot 2^{n-1} \)

ii) \( C_0 + 2 C_1 + 3 C_2 + \ldots + (n+1) C_n = (n+2) \cdot 2^{n-1} \)

iii) \( C_0 + 3 C_1 + 5 C_2 + \ldots + (2n+1) C_n = (n+1) \cdot 2^n \)

(6+6+6)

7. a) Find the equation of the straight line passing through the intersection of \( 4x - 3y - 1 = 0 \) and \( 2x - 5y + 3 = 0 \) and

i) Parallel to \( 4x + 5y = 6 \)

ii) Perpendicular to \( 2x + 3y = 12 \)

b) Explain following:

i) Polar coordinates

ii) Fundamental theorem of calculus

iii) L. Hospital’s rule

iv) Applications of Eigen values

v) Conics and their classification

(9+9)