## BE10-R4: APPLIED OPERATIONS RESEARCH

## NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.
Time: 3 Hours
Total Marks: 100
3. 

a) A company is making two products $A$ and $B$. The costs of producing one unit of $A$ and $B$ are Rs 60 and Rs 80, respectively. As per the agreement, the company has to supply at least 200 units of $B$ to its regular customers. One unit of $A$ requires one machine hour whereas $B$ has abundantly available machine hours within the company. Total machine hours available for $A$ are 400 hours. One unit of each A and B requires one labour hour each and a total of 500 labour hours is available. The company wants to minimize the total cost of production by satisfying the given requirements. Formulate the problem as a L.P.P.
b) The computer centre has got three expert programmers. The centre needs three application programs to be developed. The Head of the computer centre estimates the computer time (in hours) required by the experts to write an application program as follows:

|  |  | Programme |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | A | B | C |
| Programmer | 1 | 120 | 100 | 80 |
|  | 2 | 70 | 90 | 110 |
|  | 3 | 110 | 140 | 120 |

Assign the programmers to the program in such a way that the total programming time is least.
c) A manufacturer has to supply his customer with 600 units of his product per year. Shortages are not allowed and the storage cost amount to Rs 0.60 per unit per year. The set up cost per run is Rs. 80. Find the optimum run size and the minimum average yearly cost.
d) Use the principle of dominance to solve the game

> B

|  | I II III IV |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| I | 3 | 2 | 4 | 0 |
| II | 3 | 4 | 2 | 4 |
| ${ }^{\text {A III }}$ | 4 | 2 | 4 | 0 |
| IV | 0 | 4 | 0 | 8 |

e) For the function

$$
f(x, y)=x^{2}+x y-y^{2}, \quad x y>0
$$

find the steepest descent direction at the point $(1,2)$.
f) Write the dual of the following linear programming problem

Maximize $\quad Z=5 x_{1}+3 x_{2}$
Subject to

$$
\begin{aligned}
& x_{1}+x_{2} \leq 2 \\
& 5 x_{1}+2 x_{2} \leq 10 \\
& 3 x_{1}+8 x_{2} \leq 12
\end{aligned}
$$

and $\quad x_{1}, x_{2} \geq 0$
g) Assume that a statue is to be erected in a village square on a stone base which is to be built on a cement concrete foundation. The statue is to be made at another place, moved to the base and erected. The various operations of the entire project are given in the following random order:
A: Make statue. B: Lift statue into place. C: Construct concrete foundation. D: Compact and level the site. E: Move statue to village square. F: Construct stone base.

Construct a network diagram.
(7x4)
2.
a) Solve the following linear programming problem by simplex method:

Maximize

$$
\mathrm{Z}=3 x_{1}+5 x_{2}+4 x_{3}
$$

Subject to

$$
\begin{array}{ll} 
& 2 x_{1}+3 x_{2} \leq 8 \\
& 3 x_{1}+2 x_{2}+4 x_{3} \leq 15 \\
& 2 x_{2}+5 x_{3} \leq 10 \\
\text { and } \quad & x_{1}, x_{2}, x_{3} \geq 0
\end{array}
$$

b) Find the initial basic feasible solution by Vogel's method for the following transportation problem. Use MODI method to determine its optimal solution.

|  | 1 | 2 | 3 | Supply |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 7 | 4 | 5 |
| 2 | 3 | 3 | 1 | 8 |
| 3 | 5 | 4 | 7 | 7 |
| 4 | 1 | 6 | 2 | 14 |
| Demand | 7 | 9 | 18 | 34 |

(9+9)
3.
a) Determine an optimum sequence to process various types of fan blades each day from the following information so as to minimize the total elapsed time:

| Types of fan <br> Blades | Number to be processed <br> each day |  | Processing times on <br> (minutes) |  | Machine B <br> (minutes) |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | 4 | 4 | 8 |  |  |
| 2 | 6 | 12 | 6 |  |  |
| 3 | 5 | 14 | 16 |  |  |
| 4 | 2 | 20 | 22 |  |  |
| 5 | 4 | 8 | 10 |  |  |
| 6 | 3 | 18 | 2 |  |  |

Also work out the total elapsed time for an optimum sequence. What is the total idle time on machine $A$ and machine $B$.
b) Solve the following integer linear programming problem by branch and bound method

$$
\begin{array}{lc}
\begin{array}{l}
\text { Maximize } \\
\text { Subject to }
\end{array} & \mathrm{Z}=x_{1}+x_{2} \\
& 3 x_{1}+2 x_{2} \leq 12 \\
& x_{2} \leq 2 \\
& x_{1}, x_{2} \geq 0 \text { and both are integers. }
\end{array}
$$

4. 

a) A TV repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If the repair sets in the order in which they came in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day, what is repairman's expected idle time each day? How many jobs are ahead of the average set just brought?
b) A department head has four jobs to be performed and three subordinates. The subordinates differ in their efficiency. The time each subordinate would take to perform the jobs is given below in the matrix. How should the head allocate the jobs, one to each man, so as to minimize the total time?

| Men |  |  |  |
| :---: | :---: | :---: | :---: |
| Jobs9 26 15 <br> 13 27 6 <br> 35 20 15 <br> 18 30 20 |  |  |  |

5. 

a) The following table gives the activities in a construction project and other relevant information:

Job Description
S Start
A Processing letter
B Dictating letter
C Order execution
D Supervision
E Dispatching
F Finish

Successor Activity
A
B
D
E 7
E 6
E 2
None 0
i) Construct a network diagram.
ii) Find critical path
iii) Find EST and LST for the given data.
b) Solve the following game and find the value of the game:

Player B

|  | II |  | III |
| :---: | :---: | :---: | :---: |
|  | 1 |  |  |
| Player A | 2 |  |  |
|  | 3 |  |  |\(\left(\begin{array}{ccc}3 \& -2 \& 4 <br>

-1 \& 4 \& 2 <br>
2 \& 2 \& 6\end{array}\right)\)
6.
a) A telephone exchange has two long distance operators. The telephone company finds that during the peak load, long distance calls arrive in a Possion fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minutes. What is the probability that a subscriber will have to wait for his long distance call during the peak hours of a day? If subscribers wait and are serviced in turn, what is the expected waiting time?
b) Kedar book binder has one printing machine, one binding machine, and the manuscripts of a number of different books. Processing times are given in the following table:

| Book | Time in Hours |  |
| :---: | :---: | :---: |
|  | Printing | Binding |
| A | 5 | 2 |
| B | 1 | 6 |
| C | 9 | 7 |
| D | 3 | 8 |
| E | 10 | 4 |

Determine the order in which books should be processed on the machines, in order to minimize the total time required.
7.
a) Find the shortest path from node 1 to other nodes in the following network using Dijkstra's algorithm. The entries along the arcs represent the distances.

b) A company uses annually 25000 units of a raw material which costs Rs. 1.25/unit. Placing an order costs Rs. 100 and carrying cost is $5 \%$ per year of the average inventory. Find the economic lot size, the total inventory cost and the total annual cost including cost of material.
(9+9)

