

Course Name: A Level (1st Sem)

Subject : Introduction to DBMS

Topic: FD – Equivalence of Functional Dependencies (Part 8)

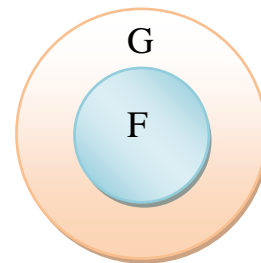
Date: 28-Apr-2020

Equivalence of Functional Dependencies

If there are given two different sets of functional dependencies for a relation, then these two may or may not be equivalent. Suppose F & G are the two sets of functional dependencies for a relational schema R, then following four cases may possible:

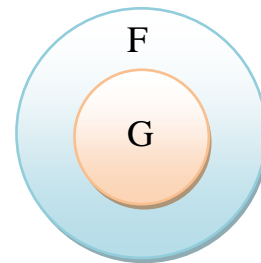
1.

$F \subseteq G$
(F is the subset of G)



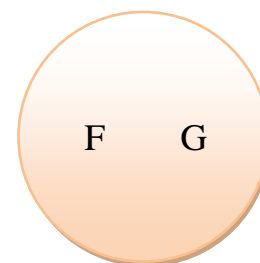
2.

$G \subseteq F$
(G is the subset of F)



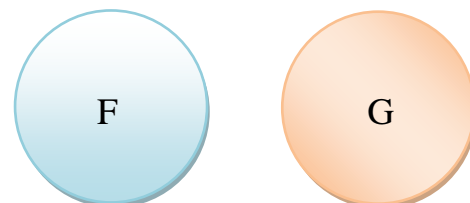
3.

$F = G$
($F \subseteq G$ and $G \subseteq F$)
(F is equivalent to G)



4.

$F \neq G$
(F is not equivalent to G)



Steps to check equivalence of two different set of functional dependencies

Suppose F & G are two set of FDs in a relational schema R:

Step 1: Find the closure of attributes that are determinant attribute (left side attributes) in FD set F, but important point is that the closures are supposed to calculate using FDs given in set G.

Step 2: Now check for each FD in F, the determinant and dependent attribute in FD set F are same as calculated in closure. If same, and since closure is calculated using FD set G, it concludes that F is subset of G ($F \subseteq G$) because G can determine all those attributes that are determined by F. If not same $F \not\subseteq G$.

Step 3: Likewise calculate whether $G \subseteq F$ or not

Using above, one of the following will come:

- I. $F \subseteq G$ but $G \not\subseteq F$
- II. $G \subseteq F$ but $F \not\subseteq G$
- III. $F \subseteq G$ and $G \subseteq F$ i.e. $F = G$
- IV. $F \not\subseteq G$ and $G \not\subseteq F$ i.e. $F \neq G$

Q 1. Suppose, a relational schema R (A, B, C) and set of functional dependencies F and G are as follow:

$$\begin{array}{ll} F : \{ A \rightarrow B, & G : \{ A \rightarrow BC, \\ B \rightarrow C, & B \rightarrow A, \\ C \rightarrow A \} & C \rightarrow A \} \end{array}$$

Check the equivalency of functional dependencies F and G.

Solution 1:

Closure of left side attribute in F Using FDs in G

$$A^+ = ABC$$

$$B^+ = BAC$$

$$C^+ = CAB$$

$F \subseteq G$ because in FD set F:

$A \rightarrow B$ and A^+ also have B,

$B \rightarrow C$ and B^+ also have C,

$C \rightarrow A$ and C^+ also have A

(A^+ , B^+ , C^+ are calculated using FD in G)

Since $F \subseteq G$ and $G \subseteq F$, **it means $A = B$**

Closure of left side attribute in G Using FDs in F

$$A^+ = ABC$$

$$B^+ = BCA$$

$$C^+ = CAB$$

$G \subseteq F$ because in FD set G

$A \rightarrow BC$ and A^+ also have BC

$B \rightarrow A$ and B^+ also have A

$C \rightarrow A$ and C^+ also have A

(A^+ , B^+ , C^+ are calculated using FD in F)

Q 2. Suppose, a relational schema R (v w x y z) and set of functional dependencies F and G are as follow:

$$F : \{ w \rightarrow x,$$

$$wx \rightarrow y,$$

$$z \rightarrow wy,$$

$$z \rightarrow v \}$$

$$G : \{ w \rightarrow xy,$$

$$z \rightarrow wx \}$$

Check the equivalency of functional dependencies F and G.

Solution 2:

Closure of left side attribute in F Using FDs in G

$$w^+ = wxy$$

$$wx^+ = wxy$$

$$z^+ = wxyz$$

F $\not\subseteq$ G because in FD set F:

$w \rightarrow x$ and w^+ also have x,

$wx \rightarrow y$ and wx^+ also have y,

$z \rightarrow wy$ and z^+ also have wy

$z \rightarrow v$ but z^+ does not have v

Closure of left side attribute in G Using FDs in F

$$w^+ = wxy$$

$$z^+ = wyvzx$$

G \subseteq F because in FD set G :

$w \rightarrow xy$ and w^+ also have xy

$z \rightarrow wx$ and z^+ also have wx

Therefore F $\not\subseteq$ G but G \subseteq F

Exercise:

1. Understand above two problems clearly and then try to solve all the exercises without referring solutions given above.

2. Suppose, a relational schema R (P,Q, R, S) and set of functional dependencies F and G are as follow:

$$F : \{ P \rightarrow Q, \\ Q \rightarrow R, \\ R \rightarrow S \}$$

$$G : \{ P \rightarrow QR, \\ R \rightarrow S \}$$

Check the equivalency of functional dependencies F and G.

