Functional Dependency – Armstrong’s Axioms Property

Armstrong’s’ axioms are set of inference rules developed by ‘William W. Armstrong’ in his 1974 paper to reason about functional dependency. These inference rules are used to derive or infer all the functional dependencies on relational database.

The property suggests rules that holds true if the following are satisfy:

- Axioms of Reflexivity
- Axioms of Augmentation
- Axioms of Transitivity

Suppose a relation R having set of attributes denoted by α, β, γ then:

**Axioms - Primary Rules (RAT)**

- **Rule 1 - Reflexivity**
  
  If β is subset of α (β ⊆ α) then α → β holds in the relation.

- **Rule 2 - Augmentation**
  
  If α → β holds in the relation and there is attribute or set of attribute γ, then γ α → γ β also holds in the relation.
  
  It means that augmenting of attribute (s) doesn’t change the functional dependency.

- **Rule 3 - Transitivity**
  
  If α → β holds and β → γ holds in the relation, then α → γ also holds in the relation.
Secondary Rules

Although the Armstrong’s axioms are sound and complete, but to simplify and ease of operations, there are some additional rules on functional dependency that are derived/concluded from Armstrong’s axioms:

- **Union Rule**
  
  If $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds in the relation, then $\alpha \rightarrow \beta \gamma$ also holds in the relation.

- **Decomposition Rule**
  
  If $\alpha \rightarrow \beta \gamma$ holds in the relation, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$ also hold in the relation.

- **Pseudo Transitivity Rule**
  
  If $\alpha \rightarrow \beta$ holds and $\gamma \beta \rightarrow \delta$ holds in the relation, then $\alpha \gamma \rightarrow \delta$ also holds in the relation.

- **Composition Rule**
  
  If $\alpha \rightarrow \beta$ holds and $x \rightarrow y$ holds in the relation, then $\alpha x \rightarrow \beta y$ also holds in the relation.

**Exercise:**

1. Why the Armstrong’ Axioms is said “sound and complete”? Explain it.
2. Proof the composition rule? How it is derived from primary rules?