Functional Dependency – Finding Candidate Key

The candidate key can be determined from given set of functional dependency in a relation.

It is an attribute or minimal set of attributes whose closure is set of all attributes in a relation.

For e.g. Suppose R (A, B, C, D) is a given relational schema, if \( AB \) is a candidate key then \( AB^+ \) must hold the all attributes of R i.e. \( AB^+ = ABCD \).

Q 1. Suppose a relational schema R(w x y z), and set of functional dependency as followings

\[
F : \{ \text{wx} \rightarrow yz, \\
\text{y} \rightarrow w, \\
\text{z} \rightarrow x \}
\]

Find the candidate keys in above relation.

Solution:

\[
\begin{align*}
\text{w}^+ &= w \\
\text{x}^+ &= x \\
\text{y}^+ &= y \text{ w} \\
\text{z}^+ &= z \text{ x} \\
\text{w x}^+ &= w \text{ x y z} \\
\text{w y}^+ &= w \text{ y} \\
\text{w z}^+ &= w \text{ z x y} \\
\text{x y}^+ &= x \text{ y z w} \\
\text{x z}^+ &= x \text{ z} \\
\text{y z}^+ &= y \text{ z w x}
\end{align*}
\]

If don’t know about how to find closure of attributes, then first refer to lecture notes of closure of attribute.
Therefore \textit{wx, wz, xy, yz are the candidate keys} in this relation because the closure of these have all the attributes of relation.

\begin{Verbatim}
Q 2. Suppose a relational schema \( R(a, b, c, d, e) \), and set of functional dependency as following
\[ F : \{ ab \rightarrow cd, \]
\[ d \rightarrow a, \]
\[ bc \rightarrow de \} \]
Find the candidate keys in above relation.
\end{Verbatim}

\begin{Verbatim}
Solution:
\[ b^+ = b \]
\[ ab^+ = a b c d e \]
\[ bc^+ = b c d e a \]
\[ bd^+ = b d a c e \]
\[ bd^+ = b e \]
\end{Verbatim}

Therefore \textit{ab, bc bd are the candidate keys} in this relation because the closure of these have all the attributes of relation.

\begin{mdframed}
\textbf{Important!}
Since, attribute ‘b’ is not at the right side of any functional dependencies, so it is the most probable chance that b should be there in the candidate key.
\end{mdframed}
Exercise:
Suppose following relational schemas and set of functional dependencies for each relations:

\[
\begin{align*}
R1 (A B C D) & & R2 (w x y z) & & R3 (P Q R S T U) \\
FD : \{ A \rightarrow B, \} & & FD : \{ wx \rightarrow yz, \} & & FD : \{ PQ \rightarrow R, \}
& & z \rightarrow w \} & & R \rightarrow S, \\
& & & & Q \rightarrow PT \}
\end{align*}
\]

Find the all candidate keys available in R1, R2 and R3.