Q. Suppose a relational schema R(w x y z), and set of functional dependency as following

\[ F : \{ x \rightarrow w, \]
\[ wz \rightarrow xy, \]
\[ y \rightarrow wxz \} \]

Find the canonical cover \( F_c \) (Minimal set of functional dependency).

Solution:

Step1: First we will check if there is any extra attribute for each FD at right side:

For this, we will decompose all the FD

\[ x \rightarrow w, \]
\[ wz \rightarrow x, \]
\[ wz \rightarrow y, \]
\[ y \rightarrow w, \]
\[ y \rightarrow x, \]
\[ y \rightarrow z \]

Now compute

\[ x^+ = xw \quad \text{(using all FDs)} \]
\[ x^+ = x \quad \text{(without using } x \rightarrow w) \]

It implies that \( x \rightarrow w \) is essential because without this FD, \( x^+ \) is different.

Like wise

\[ wz^+ = wzxy \quad \text{(using all FDs)} \]
\[ wz^+ = wzyx \quad \text{(without using } wz \rightarrow x) \]

It implies that \( wz \rightarrow x \) is not essential because without this FD, \( wz^+ \) is same.
\[ wz^+ = wz \quad \text{(without using } wz \rightarrow y) \]

It implies that \( wz \rightarrow y \) is **essential** because without this FD, \( wz^+ \) is different.

\[
\begin{align*}
y^+ &= ywzx \quad \text{(using all FDs)} \\
y^+ &= yxzw \quad \text{(without } y \rightarrow w) \\
y^+ &= yz \quad \text{(without } y \rightarrow x) \\
y^+ &= yxw \quad \text{(without } y \rightarrow z) \\
\end{align*}
\]

It implies that \( y \rightarrow w \) is **not essential** whereas \( y \rightarrow x \) and \( y \rightarrow z \) are **essential**.

**Note:** Once the non essential FD is identified, then do not include that non essential FD while computing the closure of attributes further. Exclude that FD immediately.

**Now FD set (all essential FDs)**

\[
\begin{align*}
x & \rightarrow w \\
wz & \rightarrow y \\
y & \rightarrow x \\
y & \rightarrow z \\
\end{align*}
\]

**Step 2:** Now we will check if there is any extra attribute at left side of FD.

For this

\[
wz \rightarrow y \quad \text{(only this FD has more than one attribute at left side, it may only contain extra attribute at left side)}
\]

\[
\begin{align*}
\text{compute:} & \quad wz^+ = wzyx \\
w^+ &= w \\
z^+ &= z \\
\end{align*}
\]

- If \( wz^+ \) and \( w^+ \) are same, it implies that \( z \) is extra in \( wz \rightarrow y \).
- Likewise if \( wz^+ \) and \( z^+ \) are same, it implies that \( w \) is extra in \( wz \rightarrow y \).

\( wz \rightarrow y \) is **essential** because \( wz^+ \) and \( w^+ \) are different, \( wz^+ \) and \( z^+ \) are different.

**Therefore, the minimal set of FD is**

\[
\begin{align*}
F_c : & \quad \{ x \rightarrow w, \\
wz & \rightarrow y, \\
y & \rightarrow xz \} \\
\end{align*}
\]

whereas

\[
\begin{align*}
F : & \quad \{ x \rightarrow w, \\
wz & \rightarrow xy, \\
y & \rightarrow wxz \} \\
\end{align*}
\]

\( x \) was extraneous.

\( w \) was extraneous.

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Exercise:
Suppose a relational schema R(P, Q, R, S), and set of functional dependency as following

\[ F : \{ P \rightarrow QR, \quad Q \rightarrow R, \quad P \rightarrow Q, \quad PQ \rightarrow R \} \]

Find the canonical cover \( F_c \) (Minimal set of functional dependency).