Database Normalization – Dependency Preserving Decomposition

Suppose R is a relational schema and F is the set of functional dependencies on R.
If R is decomposed into relations R1, R2, … … … Rn , each holding functional dependencies F1, F2, … … … Fn respectively.

We can say, \( F^* = F_1 \cup F_2 \cup … \cup F_n \)

Now this decomposition will be considered as dependency preserving decomposition if and only if

Every dependency in F is logically implied by \( F^* \) i.e. \( F^* = F^+ \)

It is obvious that \( F_1 \subseteq F^+ \), \( F_2 \subseteq F^+ \), and so on.

If we verify that \( F^* \) is satisfied in R, we have verified that decomposition is dependency preserving decomposition i.e. \( F_1 \cup F_2 = F \)

Suppose \( R (A \ B \ C \ D) \) and set of functional dependencies

\[ F: AB \rightarrow CD \]
\[ D \rightarrow A \]

If R is decomposed into following two relations

R1 (A D),
R2 (B C D)

Identify that this decomposition is dependency preserving or not?
Solution:

R \ (A \ B \ C \ D) \\
F: \ AB \rightarrow CD  \\
    \ D \rightarrow A

R1 \ (A \ D)
Since there are two attributes in R1 i.e. A, D 
so find \( A^+ \), \( D^+ \)
[Closure must be calculated using FD set in R]

\( A^+ = A \) [It concludes trivial FD \( A \rightarrow A \)]
\( D^+ = AD \) [It concludes FD \( D \rightarrow A \)]

The closure of attributes concludes, following FD exist in R1
F1 \{ D \rightarrow A \}

R2 \ (B \ C \ D)
Since there are three attributes in R2 i.e. B, C, D 
so find \( B^+, C^+, D^+, BC^+, CD^+, BD^+ \)
[Closure must be calculated using FD set in R]

\( B^+ = B \) [trivial]
\( C^+ = C \) [trivial]
\( D^+ = DA \) [It concludes FD, \( D \rightarrow A \) but it is invalid FD for R2 because A is not attribute in R2]
\( BC^+ = BC \) [trivial]
\( CD^+ = CDA \) [It concludes FD, \( CD \rightarrow A \) but it is invalid FD for R2 because A is not attribute in R2]
\( BD^+ = BDAC \) [It concludes FD, \( BD \rightarrow C \) A is not included at right side of FD because A is not attribute in R2]

The closure of attributes concludes, following FD exist in R2
F2 \{ BD \rightarrow C \}

Now \( F^+ = F1 \cup F2 = \{ D \rightarrow A, BD \rightarrow C \} \)
and original \( F \) in R = \{ AB \rightarrow CD, D \rightarrow A \}
It can be seen that while decomposing, the functional dependency $AB \rightarrow CD$ has been lost.

To understand, calculate $AB^+ = AB$ [ Using FD in $F^-$ ] while in $F$, $AB$ can determine $CD$ but this dependency is lost in $F^-$. Therefore, this decomposition is not dependency preserving decomposition.

**Exercise:**

Suppose $R (A \ B \ C \ D)$ and set of functional dependencies

- $F$: $AB \rightarrow C$
  - $C \rightarrow D$
  - $D \rightarrow A$

If $R$ is decomposed into following two relations

- $R_1 (A \ B \ C)$,
- $R_2 (C \ D)$

Identify that this decomposition is dependency preserving or not?